**INDIAN STATISTICAL INSTITUTE, BANGALORE CENTRE**
**B.MATH - Third Year, 2018-19**
**Statistics - IV, Midterm Examination, February 23, 2019**
**Marks are shown in square brackets.** **Maximum Marks: 50**

**1.** Consider an $I \times J$ contingency table where the $(i, j)$ cell has probability $p_{ij} > 0$ for $1 \leq i \leq I$ and $1 \leq j \leq J$. Show that the row and column factors are independent if and only if

$$\frac{p_{11}p_{ij}}{p_{1j}p_{i1}} = 1 \text{ for all } i \neq 1 \text{ and } j \neq 1. \qquad [10]$$

**2.** Suppose $X_1$ and $X_2$ are i.i.d. continuous random variables, and $U$ be any nonnegative continuous random variable independent of $X_1$. Let $Y = X_1 + U$.
(a) Show that $Y$ is stochastically larger than $X_2$.
(b) Hence, or otherwise show that a $\chi^2_{k+1}$ random variable is stochastically larger than a $\chi^2_k$ random variable. [15]

**3.** Suppose $(X_1, Y_1), (X_2, Y_2), \ldots, (X_n, Y_n)$ are i.i.d. observations from a continuous bivariate distribution with c.d.f. $F$ such that $F(u, v) = F(v, u)$ for all $u$ and $v$. Let $I_j$ be the indicator variable which is defined as $I_j = 1$ if $Y_j - X_j \geq 0$ and 0 otherwise, for $1 \leq j \leq n$. Show that $(|Y_1 - X_1|, |Y_2 - X_2|, \ldots, |Y_n - X_n|)$ and $(I_1, I_2, \ldots, I_n)$ are independently distributed. [10]

**4.** Let $X_1, X_2, \ldots, X_n$ be i.i.d. from a continuous distribution with c.d.f. $F$. Let $F_n$ denote the empirical distribution function, and let $X_{(1)} < X_{(2)} < \ldots < X_{(n)}$ be the order statistics. Then show the following.
(a) $F_n(x)$ converges in probability to $F(x)$ for each $x$ as $n \to \infty$;
(b) $D_n = \sup_{-\infty < x < \infty} |F_n(x) - F(x)|$ converges in probability to 0 as $n \to \infty$;
(c) $F(X_{(j)}) - \frac{j}{n}$ converges in probability to 0 as $n \to \infty$ for any fixed $j$. [15]